Name solutions

March 8, 2012

ECE 311 Exam 2 Spring 2012

Closed Text and Notes

1) Be sure you have 10 pages and the additional pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(12 pts).1. Fill in the table with the standard units for the following

magnetic flux density, **B**	$\frac{Wb}{m^2}$ or T
Magnetic field intensity, **H**	A/m
Electric Field Intensity, **E**	V/m
Electric Flux Density, **D**	C/m^2
Electric flux, Ψ	C
Magnetic flux, Ψ	Wb or Vs

(12 pts) 9. Indicate whether the following statements are true or false.

If you break a permanent magnet in half, one piece will be the north pole and the other piece the south pole.	True	False
The dielectric strength is the maximum electric field that a dielectric can tolerate or withstand without electric breakdown.	True	False
An isolated conducting sphere will not have a capacitance.	True	False
When a voltage is applied across a conductor, the conduction electrons will continuously accelerate from one end of the conductor to the other.	True	False
The electric flux density is due to free charges, the electric field intensity is due to free and bound charges, and the polarization is due to bound charges.	True	False
The force on a current carrying wire in a magnetic field is always perpendicular to the direction of current flow	True	False

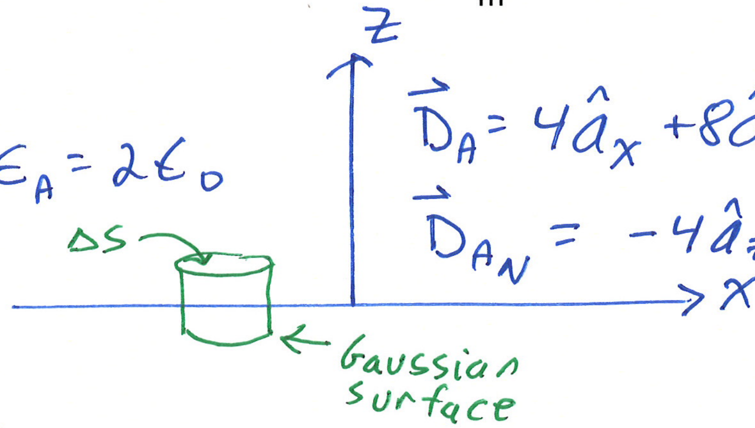
(5 pts) 3. An infinite wire on the x-axis carries a current of 2 A. What is the value of $\oint \mathbf{B} \cdot \mathbf{ds}$ over the surface of a sphere of radius 2 m centered at the origin?

0

(9 pts) 4. For region A, z > 0, $\varepsilon_A = 2\varepsilon_o$ and $\mathbf{D}_A = 4\,\hat{\mathbf{a}}_x + 8\,\hat{\mathbf{a}}_y - 4\,\hat{\mathbf{a}}_z\ \frac{C}{m^2}$. On the z = 0 plane is a sheet charge density of $\rho_s = 2\frac{C}{m^2}$. Find $\mathbf{D}_B$ in region B, where z< 0 and $\varepsilon_B = 4\varepsilon_o$.

A

$\epsilon_A = 2\epsilon_o$



$\vec{D}_A = 4\hat{a}_x + 8\hat{a}_y - 4\hat{a}_z\ \frac{C}{m^2}$

$\vec{D}_{AN} = -4\hat{a}_z\ \frac{C}{m^2}$ $\quad \vec{D}_{AT} = 4\hat{a}_x + 8\hat{a}_y\ \frac{C}{m^2}$

B

$\epsilon_B = 4\epsilon_o$

$$\vec{E}_{BT} = \vec{E}_{AT} = \frac{\vec{D}_{AT}}{2\epsilon_o} = \frac{(4\hat{a}_x + 8\hat{a}_y)\ \frac{C}{m^2}}{2\epsilon_o}$$

$$\vec{D}_{BT} = 4\epsilon_o \vec{E}_{BT} = \frac{4\epsilon_o}{2\epsilon_o}(4\hat{a}_x + 8\hat{a}_y)\frac{C}{m^2}$$

$$= 8\hat{a}_x + 16\hat{a}_y\ \frac{C}{m^2}$$

Applying Gauss' law to the Gaussian surface shown

$$\oint \vec{D}\cdot d\vec{s} = D_{AN}\Delta S - D_{BN}\Delta S = \rho_s \Delta S$$

$$D_{AN} - D_{BN} = \rho_s$$

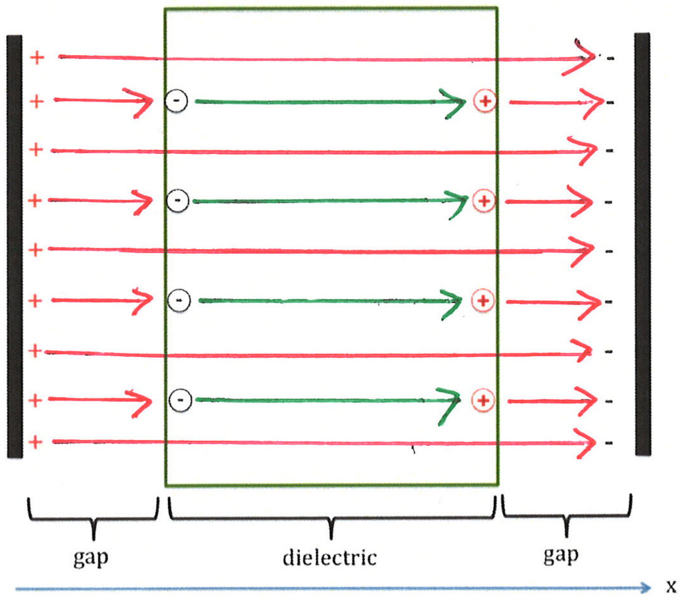
$$\frac{C}{m^2} - D_{BN} = 2\frac{C}{m^2} \quad \Rightarrow \quad D_{BN} = -6\frac{C}{m^2}$$

$$\vec{D}_B = 8\hat{a}_x + 16\hat{a}_y - 6\hat{a}_z\ \frac{C}{m^2}$$

(12 pts) 5. Shown is a parallel plate capacitor. Part of the region between the plates is filled with a dielectric. A voltage is applied and the resulting charge density on the positive capacitor plate is $\rho_S = 5(\frac{10^{-9}}{36\pi})\frac{C}{m^2}$.

There is a resulting bound charge density of the surface facing the positive plate of $\rho_B = (\frac{10^{-9}}{36\pi})\frac{C}{m^2}$.

Find the electric field intensity, the electric flux density and the polarization in the dielectric and in the free space gaps inside the capacitor.



The only free charge is on the plates. So the electric flux density in the gaps and dielectric will be

$$\vec{D} = 5\left(\frac{10^{-9}}{36\pi}\right)\hat{a}_x \ \frac{C}{m^2}$$

The Polarization is due to bound charge so in the dielectric

$$\vec{P} = \left(\frac{10^{-9}}{36\pi}\right)\hat{a}_x \ \frac{C}{m^2}$$

So in the two gaps

$$\vec{D} = 5\left(\frac{10^{-9}}{36\pi}\right)\hat{a}_x \ \frac{C}{m^2}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{5\left(\frac{10^{-9}}{36\pi}\right)\frac{C}{m^2}}{\left(\frac{10^{-9}}{36\pi}\right)\frac{F}{m}}\hat{a}_x = 5\hat{a}_x \frac{V}{m}$$

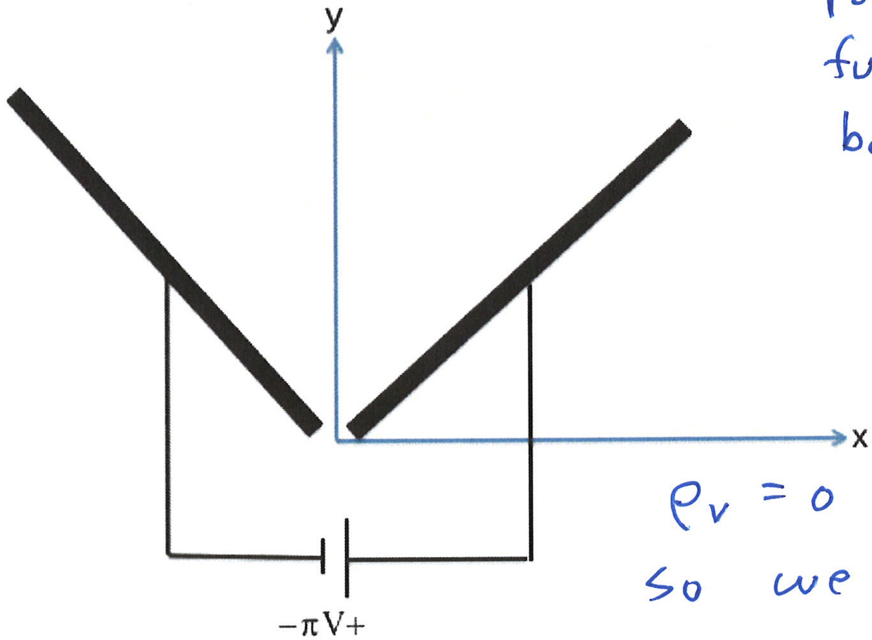
$$\vec{P} = 0$$

in the dielectric

$$\vec{D} = 5\left(\frac{10^{-9}}{36\pi}\right)\hat{a}_x \ \frac{C}{m^2} \qquad \vec{P} = \left(\frac{10^{-9}}{36\pi}\right)\hat{a}_x \ \frac{C}{m^2}$$

$$\vec{E} = \frac{\vec{D} - \vec{P}}{\epsilon_0} = \frac{\left[5\left(\frac{10^{-9}}{36\pi}\right) - \left(\frac{10^{-9}}{36\pi}\right)\right]\frac{C}{m^2}}{\left(\frac{10^{-9}}{36\pi}\right)\frac{F}{m}}\hat{a}_x = 4\hat{a}_x \frac{V}{m}$$

(10 pts) 6. Shown are two conductors. One is the semi-infinite half-plane at $\phi=\frac{\pi}{4}$ and the other is the semi-infinite half-plane at $\phi=\frac{3\pi}{4}$. Find the potential and electric field intensity for $\frac{\pi}{4}<\phi<\frac{3\pi}{4}$. The region $\frac{\pi}{4}<\phi<\frac{3\pi}{4}$ is free space.



from symmetry the potential is only a function of ϕ with boundary conditions

$$V\left(\frac{\pi}{4}\right) = \pi \text{ V}$$

$$V\left(\frac{3\pi}{4}\right) = 0$$

$\rho_v = 0$ for $\frac{\pi}{4} < \phi < \frac{3\pi}{4}$ so we can use Laplace's eq.

$$\nabla^2 V = 0 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

since $V(\phi)$ $\frac{\partial V}{\partial \rho} = \frac{\partial V}{\partial z} = 0$ so

$$\frac{1}{\rho}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$\frac{\partial V}{\partial \phi} = A$$

$$V(\phi) = A(\phi) + B$$

apply the boundary conditions

$$V\left(\frac{3\pi}{4}\right) = 0 = A\frac{3\pi}{4} + B$$

$$B = -A\frac{3\pi}{4}$$

$$V(\phi) = A\left(\phi - \frac{3\pi}{4}\right)$$

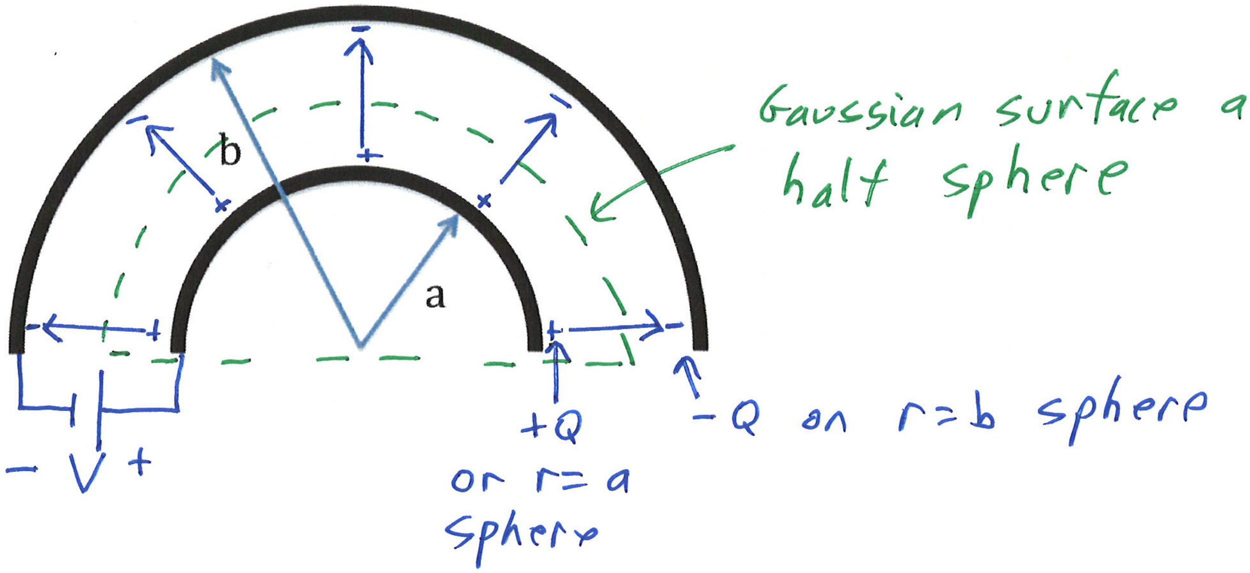
$$V\left(\frac{\pi}{4}\right) = \pi \text{ V} = A\left(\frac{\pi}{4} - \frac{3\pi}{4}\right)$$

$$A = -2 \text{ V}$$

$$V(\phi) = (-2\text{V})\left(\phi - \frac{3\pi}{4}\right)$$

$$V(\phi) = \left(\frac{3\pi}{2} - 2\phi\right) \text{V}$$

(10 pts) 7. Neglect fringing fields and find the capacitance between the curved conductors at r = a and r = b shown below. The curved conductors are half spheres. The region between the surfaces is free space.



Use Gauss' Law to determine D for $a < r < b$

$$\oint \vec{D} \cdot \vec{ds} = D_r(r) \frac{1}{2} 4\pi r^2 = Q$$

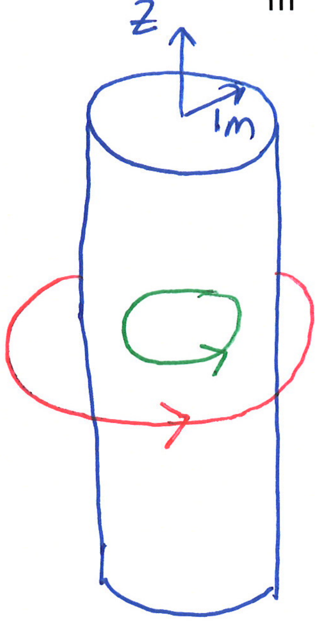
$$D_r(r) = \frac{Q}{2\pi r^2} \qquad E(r) = \frac{Q}{2\pi \epsilon_0 r^2}$$

$$V = -\int_b^a \vec{E} \cdot \vec{d\ell} = -\int_b^a \frac{Q}{2\pi\epsilon_0 r^2} \hat{a}_r \cdot dr \hat{a}_r$$

$$V = -\frac{Q}{2\pi\epsilon_0} \int_b^a \frac{dr}{r^2} = \frac{Q}{2\pi\epsilon_0} \frac{1}{r} \Big|_b^a = \frac{Q}{2\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{2\pi\epsilon_0 ab}{b - a}$$

(10 pts) 8. An infinitely long cylinder of radius 1m is centered on the z-axis. A current density of $\mathbf{J} = 1\,\frac{\text{A}}{\text{m}^2}\,\hat{\mathbf{a}}_z$ is flowing in the cylinder. Determine the magnetic field intensity everywhere.



From symmetry $\vec{H} = H_\phi(\rho)\,\hat{a}_\phi$

We will apply Ampere's circuital law in two regions as shown, for $\rho < 1\text{m}$ and for $\rho > 1\text{m}$

for $\rho < 1\text{m}$

$$\oint \vec{H}\cdot d\vec{l} = I_{encl} = J_z A$$

$$\int_0^{2\pi} H_\phi(\rho)\,\hat{a}_\phi \cdot \rho\, d\phi\, \hat{a}_\phi = \left(1\tfrac{A}{m^2}\right)\pi\rho^2$$

$$\rho\, H_\phi(\rho)\int_0^{2\pi} d\phi = H_\phi(\rho)\, 2\pi\rho = \left(1\tfrac{A}{m^2}\right)\pi\rho^2$$

$$\vec{H} = \frac{\rho}{2}\,\hat{a}_\phi$$

for $\rho > 1\text{m}$

$$\oint \vec{H}\cdot d\vec{l} = \int_0^{2\pi} H_\phi(\rho)\,\hat{a}_\phi\cdot\rho\, d\phi\,\hat{a}_\phi = 2\pi\rho\, H_\phi(\rho) = \left(1\tfrac{A}{m^2}\right)\pi(1\text{m})^2$$

$$H_\phi(\rho) = \frac{1}{2\rho}$$

$$\vec{H} = \frac{1}{2\rho}\,\hat{a}_\phi$$

(8 pts) 9 . A 1000 m long wire of cross-sectional area 1 m^2 has 1 A flowing in it when a battery of 1 V is connected across it.

(4 pts) what is the electric field intensity in the wire?

$$E = \frac{V}{L} = \frac{1\,V}{1000\,m} = 10^{-3}\,\frac{V}{m}$$

(4 pts) What is the he resistivity of the wire?

$$J = \frac{I}{S} = \frac{1\,A}{1\,m^2} = 1\,\frac{A}{m^2}$$

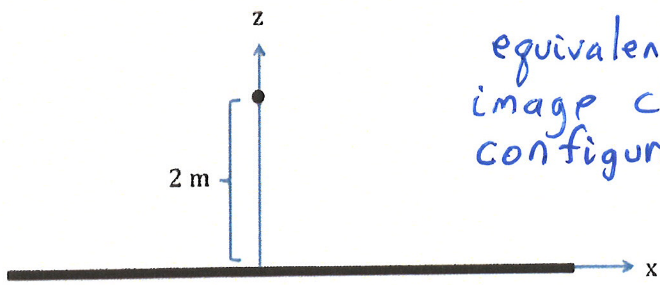
$$J = \sigma E = \frac{1}{\rho} E$$

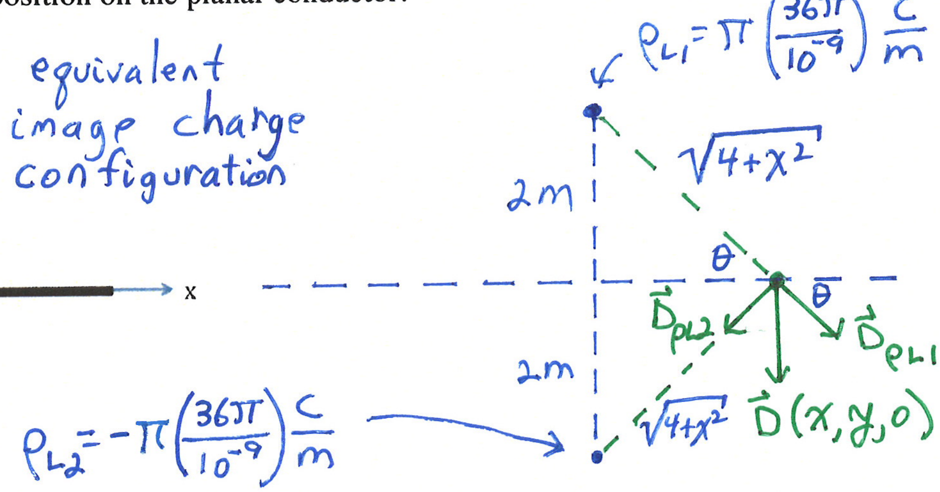
$$1\,\frac{A}{m^2} = \frac{1}{\rho}\,10^{-3}\,\frac{V}{m}$$

$$\rho = \frac{10^{-3}\,\frac{V}{m}}{1\,\frac{A}{m^2}} = 10^{-3}\,\frac{V}{A}\,m$$

$$\rho = 10^{-3}\,\Omega\,m$$

(12 pts) 10. An infinitely long, uniform line charge of $\rho_l = \pi(\frac{36\pi}{10^{-9}})\frac{C}{m}$ is located a distance 2 m above an infinite conducting plane, the z = 0 plane, and parallel to the y-axis. Find the surface charge density as a function of position on the planar conductor.





on the z=0 plane, there is just an electric flux density component in the $-\hat{a}_z$ direction because the x-components from the ρ_{L1} and ρ_{L2} line charges will cancel.

The magnitude of the electric flux density a distance d from a line charge ρ_L is $D = \frac{\rho_L}{2\pi d}$

so $|\vec{D}_{\rho L1}| = |\vec{D}_{\rho L2}| = \frac{\pi\left(\frac{36\pi}{10^{-9}}\right)}{2\pi\sqrt{4+x^2}}\frac{C}{m^2}$

$$\vec{D}_{\rho L1 z} = \frac{-\left(\frac{36\pi}{10^{-9}}\right)}{2\sqrt{4+x^2}}\sin\theta\,\hat{a}_z\frac{C}{m^2} = -\frac{\left(\frac{36\pi}{10^{-9}}\right)}{2\sqrt{4+x^2}}\frac{2}{\sqrt{4+x^2}}\hat{a}_z\frac{C}{m^2}$$

$$= -\frac{\left(\frac{36\pi}{10^{-9}}\right)}{4+x^2}\hat{a}_z\frac{C}{m^2}$$

so $$\vec{D}(x,y,0) = \frac{-2}{4+x^2}\left(\frac{36\pi}{10^{-9}}\right)\hat{a}_z\frac{C}{m^2}$$

and

$$\rho_S(x,y,0) = \frac{-2}{4+x^2}\frac{36\pi}{10^{-9}}\frac{C}{m^2}$$